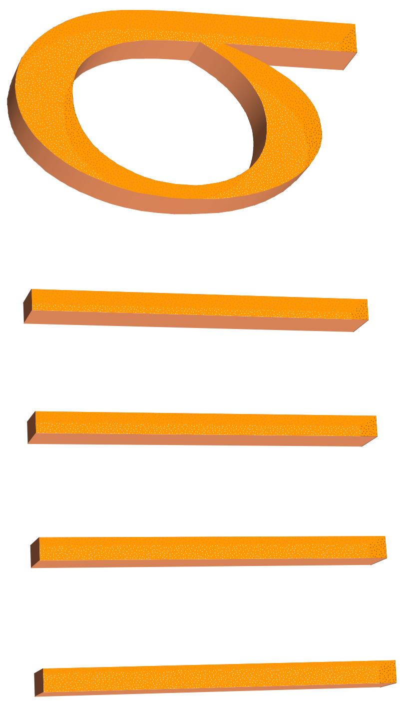
Board Breaking
The hand is quicker than the



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Abstract

Why can I push on a board all day but it only breaks if I strike it at high speed? The question of the why we get different outcomes using the same stress/force at different times of "application" led to modeling wood, or more specifically the anisotropic tensor associated with it.

Board Breaking

Breaking a board with a part of the human body is an incredible sight to behold. After seeing a feat like this several question arise, one of which is: Why can a board have a force applied to it over a long period of time and remain intact but when it receives the same force in a shorter time it breaks.? In exploring this question the fields of martial arts, physics, bio-mechanics, metaphysics and continuum mechanics all come up. We will cover some basic background of Martial arts and physics to better understand the model of wood we arrive at using the principles of continuum mechanics.

1.1 The art (Martial) of board breaking

First off, we have the question why break a board? In many martial arts, board breaking is used as a way of demonstrating the use of proper technique and of focus, not (as may be assumed) to show off strength. As Sihak Henry Cho, grand master at the Karate Institute puts it. ". "Rather than seeing students break a dozen boards, I'd like to see them jump over my shoulder and break one board while flying through the air." Now that we know why someone would break a board we should explore what is nessary, from a martial arts viewpoint to break a board. The three main principles are focus, training, and speed. These are interdependant qualities (Fig 1). Focus is nessary to maintain good training, through training speed is increased, with more speed comes the need to focus for control. i.e.

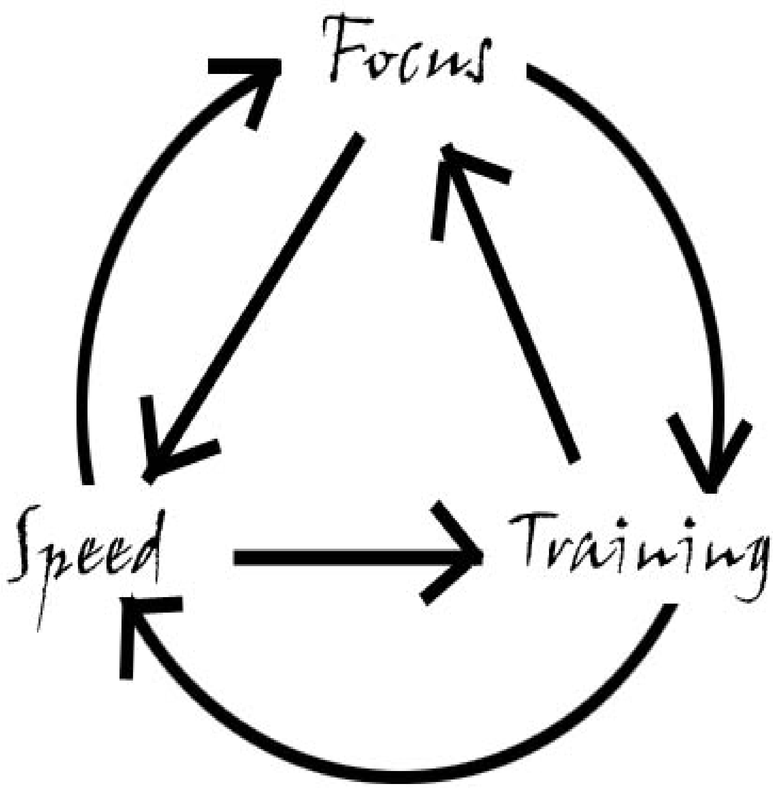


Fig. 1

Training increases speed by the phenomenon of automatic muscle memory (AMM). The more times the human body repeats a movement the quicker that movement becomes. In martial arts training the same punches and kicks are performed over and over so the body becomes accustomed to moving in those ways. Now the brain does not have to regulate a complex set of muscle movements, through AMM the body "remembers" what to do on its own.

Indeed practice makes perfect. When martial artists are asked how they can break boards with their hands, the majority respond with something like, "the trick is to concentrate through the board." This semi-mystical statement is actually describing a way of maximizing the speed of their strike. If we think back to the physics of a pendulum (Fig. 2), the mystic will reveal it's self.

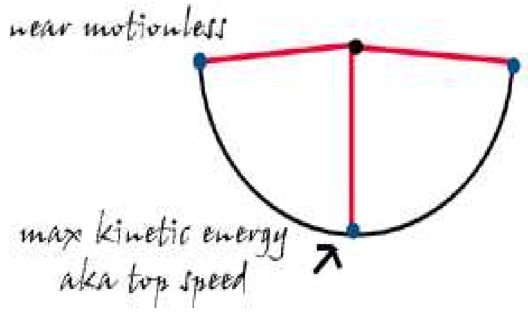


Fig 2

If we hold a pendulum up we increase its potential energy. When we let it go the potential energy is converted to kinetic energy by gravity. However once the pendulum passes the lowest point of its swing gravity begins to slow it down. Now its kinetic energy is turning back into potential energy until it stops and begins to swing back the other way repeating the process. What does this have to do with a punch? Examining a punch we observe that the fist moves away from the body until the arm is fully extended, and then the arm retracts and returns the fist to the body. If we think of the human arm as the pendulum it become clear that at full extension of the arm a punch is not moving (speaking as instantaneous velocity)

As reported in Discover magazine. [20]

> Jearl Walker, a former tae kwon do student who now teaches physics at Cleveland State University, set up a study much like Feld's and McNair's. A well-thrown fist, he found, reaches its maximum velocity when the arm is about 80 percent extended. "That's exactly what my tae kwon do master

> had taught me," Walker says. "You learn to focus your punch in your imagination so that it terminates inside your opponent's body, rather than on the surface. To deliver the maximum power, you want to make contact before the slowdown begins.

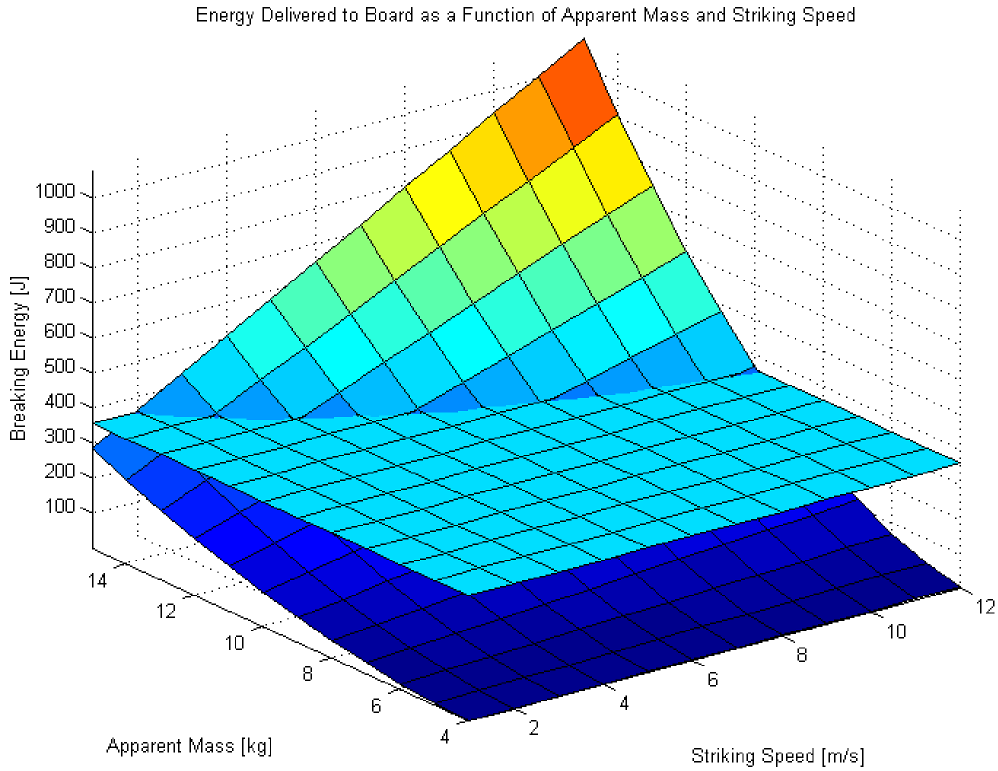
Speed also comes from the martial arts techniques them selves. The very moves that are practiced into AMM have been refined over hundreds of years to achieve maximum efficiency.

> "Amazingly, there are no tricks involved at all," says Michael Feld, a physicist at MIT. "What you have here is one of the most efficient human movements ever conceived. We've found nothing in our studies to improve upon the art." [20]

Looking to the work of Bill Pottle [12] we can see a direct relation of speed and technique in the ability to break a board.

> Figure 3 shows a 3D representation of the breaking energy applied to the board as a function of apparent mass (m_a) and striking speed. Apparent mass is a function of technique. For a punch, someone could punch with only their arm, or they could put their shoulder, hip, or whole body into the strike. Striking speed is defined as the instantaneous velocity of the limb at the point of contact with the board. The base case is a fresh pine board with a volume of 0.0017 m^3. With M_r=0.061 GN/m^2 and E=8.81 GN/m^2, the breaking energy is 359 J. The energy required to break the board is shown as a horizontal plane with z=359 J. Any combination of striking speed and apparent mass above this plane will result in fracture; any combination below the plane will result in a bruised limb.

Fig. 3



It should be noted in this paper we are assuming that the board in question is being held fixed at two of the ends (Fig. 4)

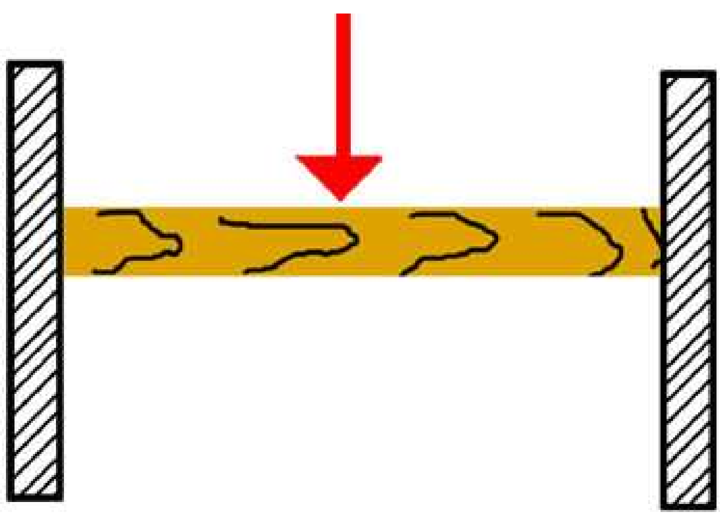
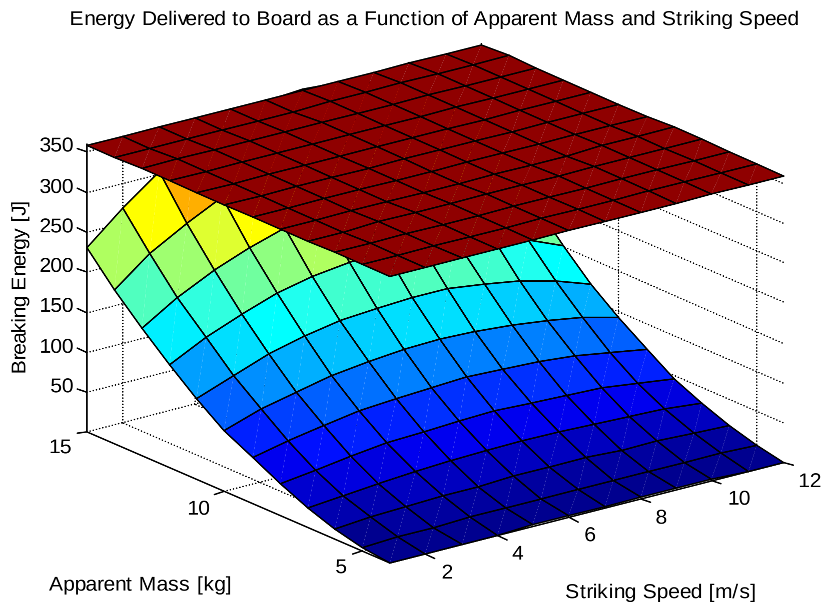


Fig. 4

A moving board adds some interesting problem/benefits to the process. If the board moves away form the strike then the impact will not occur at maximum velocity.

(Fig. 5) The energy delivered to a board is a function of striking speed and apparent mass for a two handed hold where the holders absorb 85% of the force. Note that now the horizontal plane covers most of the breaking energy curve, signifying a near impossible break. [12]

Fig. 5



On the other ha

easier. Martial arts masters will use this fact to help small children in board breaking.

1.2 Modeling the wood

Now that we have covered breaking from the martial aspect it is time to look at modeling the wood. Starting with Hooke's Law, (which is covered in engineering and continuum mechanics courses). Hooke's Law relates Stress (t_{ij}) linearly (σ_{ijkl}) to strain (e_{kl}).

$$t_{ij} = \sigma_{ijkl} \, e_{kl}. \quad (1.2.1)$$

In order to get a model of wood we need to measure the components of σ_{ijkl} (these are constitutive equations that describe a material). This fourth order tensor has 81 components. Measuring this many coefficients is difficult at best. However we can exploit some of the information we have about the stress and strain tensors to remove some of the components. From earlier work in continuum mechanics we know that t_{ij} and e_{kl} are symmetric tensors, this implies that σ_{ijkl} must have minor symmetries. Making use of a different notation we will write second order tensors as column vectors (1 X 9) and fourth order tensors as matrices (9 X 9).

$$\begin{pmatrix} t_{11} \\ t_{12} \\ t_{13} \\ t_{21} \\ t_{22} \\ t_{23} \\ t_{31} \\ t_{32} \\ t_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{1111} & \sigma_{1112} & \sigma_{1113} & \sigma_{1121} & \sigma_{1122} & \sigma_{1123} & \sigma_{1131} & \sigma_{1132} & \sigma_{1133} \\ \sigma_{1211} & \sigma_{1212} & \sigma_{1213} & \sigma_{1221} & \sigma_{1222} & \sigma_{1223} & \sigma_{1231} & \sigma_{1232} & \sigma_{1233} \\ \sigma_{1311} & \sigma_{1312} & \sigma_{1313} & \sigma_{1323} & \sigma_{1322} & \sigma_{1323} & \sigma_{1331} & \sigma_{1332} & \sigma_{1333} \\ \sigma_{2111} & \sigma_{2112} & \sigma_{2113} & \sigma_{2121} & \sigma_{2122} & \sigma_{2123} & \sigma_{2131} & \sigma_{2132} & \sigma_{2133} \\ \sigma_{2211} & \sigma_{2212} & \sigma_{2213} & \sigma_{2221} & \sigma_{2222} & \sigma_{2223} & \sigma_{2231} & \sigma_{2232} & \sigma_{2233} \\ \sigma_{2311} & \sigma_{2312} & \sigma_{2313} & \sigma_{2321} & \sigma_{2322} & \sigma_{2323} & \sigma_{2331} & \sigma_{2332} & \sigma_{2333} \\ \sigma_{3111} & \sigma_{3112} & \sigma_{3113} & \sigma_{3121} & \sigma_{3122} & \sigma_{3123} & \sigma_{3131} & \sigma_{3132} & \sigma_{3133} \\ \sigma_{3211} & \sigma_{3212} & \sigma_{3213} & \sigma_{3221} & \sigma_{3222} & \sigma_{3223} & \sigma_{3231} & \sigma_{3232} & \sigma_{3233} \\ \sigma_{3311} & \sigma_{3312} & \sigma_{3313} & \sigma_{3321} & \sigma_{3322} & \sigma_{3323} & \sigma_{3331} & \sigma_{3332} & \sigma_{3333} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{12} \\ e_{13} \\ e_{21} \\ e_{22} \\ e_{23} \\ e_{31} \\ e_{32} \\ e_{33} \end{pmatrix}$$

(1.2.2)

Using $t_{ij} = t_{ji}$ implies $\sigma_{ij\,kl} = \sigma_{ji\,kl}$ i.e. $t_{12} = t_{21}$ gives us $\sigma_{12\,kl} = \sigma_{21\,kl}$ which "kills" three rows, (3X 9 = 27 components gone)

$$\begin{pmatrix} \sigma_{1111} & \sigma_{1112} & \sigma_{1113} & \sigma_{1121} & \sigma_{1122} & \sigma_{1123} & \sigma_{1131} & \sigma_{1132} & \sigma_{1133} \\ \sigma_{1211} & \sigma_{1212} & \sigma_{1213} & \sigma_{1221} & \sigma_{1222} & \sigma_{1223} & \sigma_{1231} & \sigma_{1232} & \sigma_{1233} \\ \sigma_{1311} & \sigma_{1312} & \sigma_{1313} & \sigma_{1323} & \sigma_{1322} & \sigma_{1323} & \sigma_{1331} & \sigma_{1332} & \sigma_{1333} \\ \sigma_{2111} & \sigma_{2112} & \sigma_{2113} & \sigma_{2121} & \sigma_{2122} & \sigma_{2123} & \sigma_{2131} & \sigma_{2132} & \sigma_{2133} \\ \sigma_{2211} & \sigma_{2212} & \sigma_{2213} & \sigma_{2221} & \sigma_{2222} & \sigma_{2223} & \sigma_{2231} & \sigma_{2232} & \sigma_{2233} \\ \sigma_{2311} & \sigma_{2312} & \sigma_{2313} & \sigma_{2321} & \sigma_{2322} & \sigma_{2323} & \sigma_{2331} & \sigma_{2332} & \sigma_{2333} \\ \sigma_{3111} & \sigma_{3112} & \sigma_{3113} & \sigma_{3121} & \sigma_{3122} & \sigma_{3123} & \sigma_{3131} & \sigma_{3132} & \sigma_{3133} \\ \sigma_{3211} & \sigma_{3212} & \sigma_{3213} & \sigma_{3221} & \sigma_{3222} & \sigma_{3223} & \sigma_{3231} & \sigma_{3232} & \sigma_{3233} \\ \sigma_{3311} & \sigma_{3312} & \sigma_{3313} & \sigma_{3321} & \sigma_{3322} & \sigma_{3323} & \sigma_{3331} & \sigma_{3332} & \sigma_{3333} \end{pmatrix}.$$

(1.2.3)

Now we will make use of the symmetry on the other side of the relation.

i.e. $e_{kl} = e_{lk}$ this implies $\sigma_{ij12} = \sigma_{ij21}$ so now we can "kill" three more rows

(3X6 = 18 more components gone)

(1.2.5) $$\begin{pmatrix} \sigma_{1111} & \sigma_{1112} & \sigma_{1113} & \sigma_{1121} & \sigma_{1122} & \sigma_{1123} & \sigma_{1131} & \sigma_{1132} & \sigma_{1133} \\ \sigma_{1211} & \sigma_{1212} & \sigma_{1213} & \sigma_{1221} & \sigma_{1222} & \sigma_{1223} & \sigma_{1231} & \sigma_{1232} & \sigma_{1233} \\ \sigma_{1311} & \sigma_{1312} & \sigma_{1313} & \sigma_{1323} & \sigma_{1322} & \sigma_{1323} & \sigma_{1331} & \sigma_{1332} & \sigma_{1333} \\ \sigma_{2211} & \sigma_{2212} & \sigma_{2213} & \sigma_{2221} & \sigma_{2222} & \sigma_{2223} & \sigma_{2231} & \sigma_{2232} & \sigma_{2233} \\ \sigma_{2311} & \sigma_{2312} & \sigma_{2313} & \sigma_{2321} & \sigma_{2322} & \sigma_{2323} & \sigma_{2331} & \sigma_{2332} & \sigma_{2333} \\ \sigma_{3311} & \sigma_{3312} & \sigma_{3313} & \sigma_{3321} & \sigma_{3322} & \sigma_{3323} & \sigma_{3331} & \sigma_{3332} & \sigma_{3333} \end{pmatrix}.$$

Expressing this idea more rigorously, we see that the mathematical expression

for the first element of $\mathbf{t}_{ij}$ is:

$$t_{11} = \sigma_{1111} e_{11} + \cdots + \sigma_{1112} e_{12} + \cdots \ \sigma_{1121} e_{21} + \cdots$$
$$= \sigma_{1111} e_{11} + \cdots + (\sigma_{1112} + \sigma_{1121}) e_{12} + \cdots$$

(Recall that $e_{lk} = e_{kt}$ so we can combine the e12 and e21 terms can be collected together.)

(1.2.4) $$= \sigma_{1111} e_{11} + \cdots + 1/2\ (\sigma_{1112} + \sigma_{1121}) 2e_{12} + \cdots$$

Also we have from the symmetry of $\mathbf{t}_{ij}$ the equations for t_{21}, t_{31}, and t_{32} are not needed.
So in general we have:

(1.2.5) $$t_{ij} = \sum_{k=1}^{3} \sum_{l=1}^{3} \sigma_{ijkl} e_{kl}$$

a.k.a. $$t_{ij} = \sigma_{ij11} e_{11} + \sigma_{ij22} e_{22} + \sigma_{ij33} e_{33} + 1/2\ (\sigma_{ij12} + \sigma_{ij21}) 2e_{12} +$$

(1.2.6) $$1/2\ (\sigma_{ij23} + \sigma_{ij32}) 2e_{23} + 1/2\ (\sigma_{ij31} + \sigma_{ij13}) 2e_{31}$$

If we let $\sigma_{ij12} = 1/2\ (\sigma_{ij12} + \sigma_{ij21})$ we can assume $\sigma_{ijkl} + \sigma_{ijll}$.

The previous work is based on the notes of Lynn Bennethum [1]

Now equation (1.2.2) can be written in the more compact form.

$$\begin{pmatrix} t_{11} \\ t_{22} \\ t_{33} \\ t_{12} \\ t_{13} \\ t_{23} \end{pmatrix} = \begin{pmatrix} \sigma_{1111} & \sigma_{1122} & \sigma_{1133} & \sigma_{1112} & \sigma_{1113} & \sigma_{1123} \\ \sigma_{2211} & \sigma_{2222} & \sigma_{2233} & \sigma_{2212} & \sigma_{2213} & \sigma_{2223} \\ \sigma_{3311} & \sigma_{3322} & \sigma_{3333} & \sigma_{3312} & \sigma_{3313} & \sigma_{3323} \\ \sigma_{1211} & \sigma_{1222} & \sigma_{1233} & \sigma_{1212} & \sigma_{12213} & \sigma_{1223} \\ \sigma_{1311} & \sigma_{1322} & \sigma_{1333} & \sigma_{1312} & \sigma_{1313} & \sigma_{1323} \\ \sigma_{2311} & \sigma_{2322} & \sigma_{2333} & \sigma_{2312} & \sigma_{2313} & \sigma_{2323} \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{22} \\ e_{33} \\ 2\,e_{12} \\ 2\,e_{13} \\ 2\,e_{23} \end{pmatrix} \tag{1.2.7}$$

(Note that the rows have been reordered into a more useful arrangement)

Now the 81 items that needed to be measured have been reduced to a mere 36 (81 – 27 – 18 = 36).

Looking to remove even more components an examination of a board may prove useful. Upon inspection we may assume a there is a homogenous quality. One part of the board looks a lot like the other, but is this always the case? Thinking about how boards are planed from a tree and the way in which a tree grows casts doubt on the board being truly isotropic. Trees grow in concentric circles (Fig. 6) and boards are cut off laterally (Fig. 7)

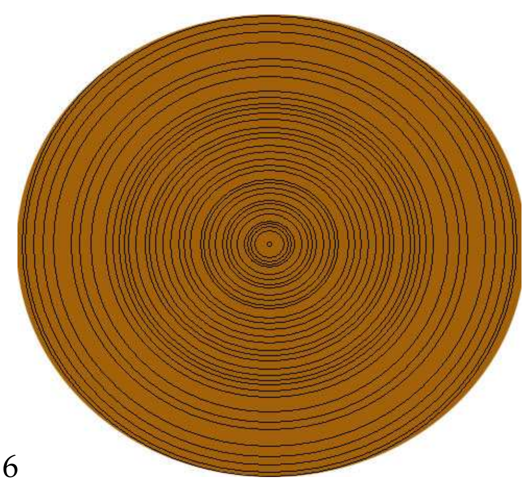


Fig. 6

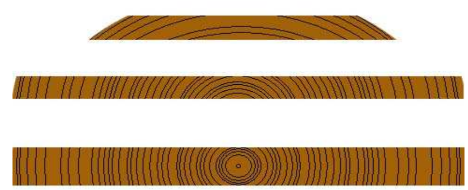


Fig.7

The rings are cut through symmetrically in the center cut only and we will see this can make a difference to someone breaking a board.

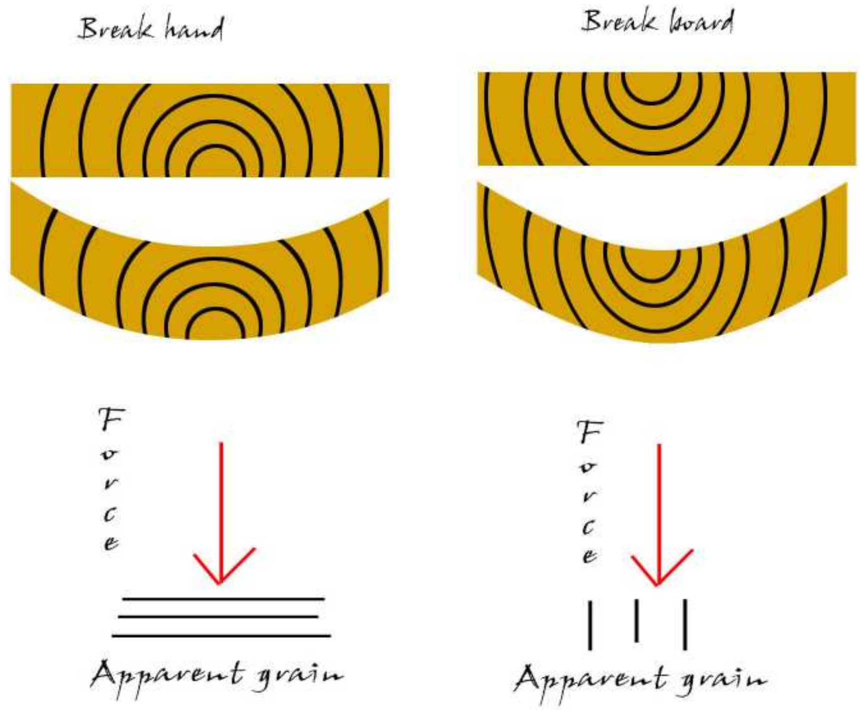


Fig. 8

In an off center cut the end grain (Circle segments) the

"cup" can point up or down. (Fig. 8) As a force begins to bend the board these arcs begin to "flatten" out to be either perpendicular or parallel to the force. A perpendicular "apparent grain" shunts the force away for the impact site making the break more difficult. When the "apparent grain" is parallel the force shoots through the grain cleaving the board more easily. However the difference in force necessary in these two cases is not large enough to frighten an experienced martial artist, (beginners should be wary). So for the rest of the paper we will assume that wood is homogeneous. Looking again to the wood to find some way of getting rid of more components we notice that the result of a force applied to the "top' and "bottom" of the board is unaffected when we rotate the board (Fig. 9). In other words if we place a mirror in the xz-plane or yz-plane we cannot tell if it is a reflection or not.

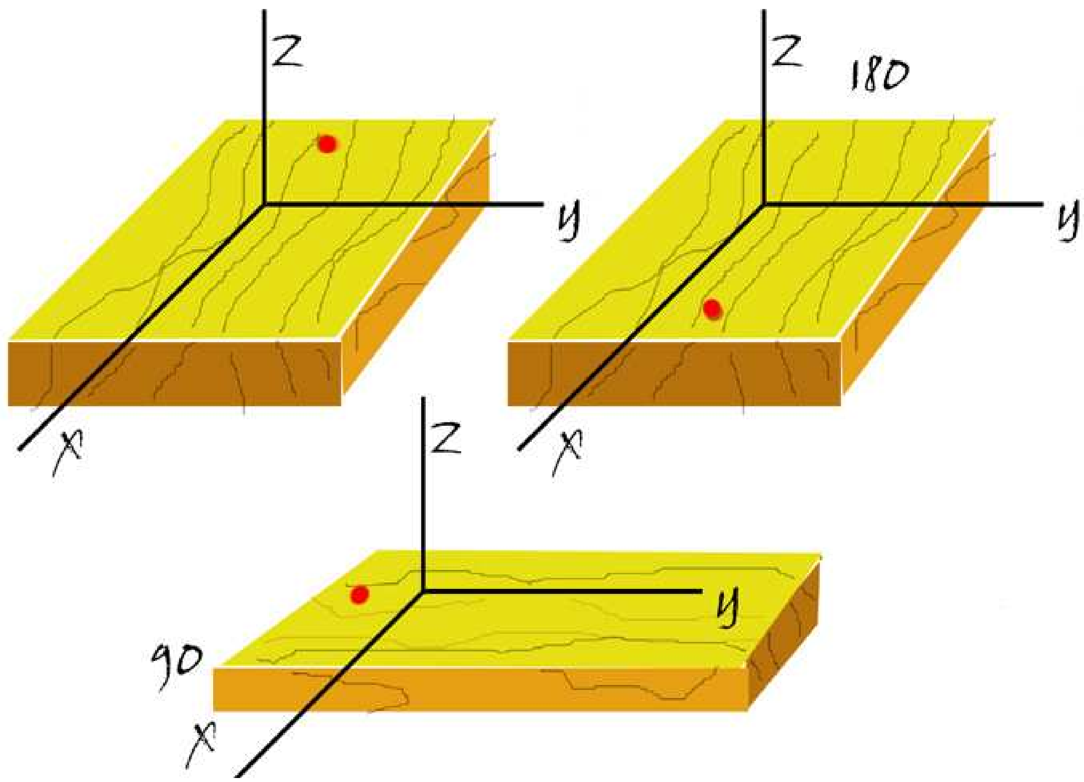


Fig. 9

As we rotate the wood in a plane we are not looking for geometric or stress state symmetry but rather directional symmetry at a point in the plane. (reflective in the coordinate not in the rotated plane) This is called a plane of elastic symmetry. George Mase states,

> A Plane of elastic symmetry exists at a point where the elastic constants have the same values for every pair of coordinate systems which are the reflected images of one another with respect to the plane. The axes of such coordinate systems are referred to as 'equivalent elastic directions.' If the x_1x_2 plane is one of elastic symmetry, the constants C_{KM} are invariant under the coordinate transformation
>
> $x_1' = x_1$, $x_2' = x_2$, $x_3' = -x_3$ …

given by the matrix $[a_{ij}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$.[14]

He further states that using this transformation on the strain and stress tensors leads to the matrix.

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & 0 & 0 & \sigma_{16} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & 0 & 0 & \sigma_{26} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 & 0 & \sigma_{36} \\ 0 & 0 & 0 & \sigma_{44} & \sigma_{45} & 0 \\ 0 & 0 & 0 & \sigma_{54} & \sigma_{55} & 0 \\ \sigma_{61} & \sigma_{62} & \sigma_{63} & 0 & 0 & \sigma_{66} \end{pmatrix} \quad (1.6)$$

If this process is repeated for the other two planes we get the much nicer looking matrix.

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & 0 & 0 & 0 \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & 0 & 0 & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{66} \end{pmatrix} \quad (1.7)$$

In his book Continuum Mechanics A.J. M. Spencer [18] states

> A material which has reflectional symmetry with respect to each of three mutually orthogonal planes is said to be orthotropic. To a good approximation, wood is an example of such a material.

To make things even nicer we can assume that wood is hyperelastic.

> Hyperelasticity ignores the thermal effects and asumes that the elastic potential function always exists, a function of the strains alone: it is a purely mechanical theory.
>
> ~Malvern ,Laurence E. intro to the Mec of a cont medium[13]

We do this so we can assume wood has strain energy function (W). This is a scalar function of one of the strain or deformation tensors, whose derivative with respect to a strain component determines the corresponding stress component. This gives us symmetry in our matrix to finally get the components to be measured down to nine.

$$\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & 0 & 0 & 0 \\ \sigma_{12} & \sigma_{22} & \sigma_{23} & 0 & 0 & 0 \\ \sigma_{13} & \sigma_{23} & \sigma_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{66} \end{pmatrix} \quad (1.8)$$

A little research found that pine has the matrix:

$$\begin{pmatrix} 1.226 & 0.753 & 0.747 & 0 & 0 & 0 \\ 0.753 & 1.775 & 0.941 & 0 & 0 & 0 \\ 0.747 & 0.941 & 17.004 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.348 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.816 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.16 \end{pmatrix} \quad (1.9)$$

Relating these numbers back to the board we see smallest value in the diagonal of the "non-shear" section is 1.226 which would be the top of the board ((Fig. 10) where we want to hit it).

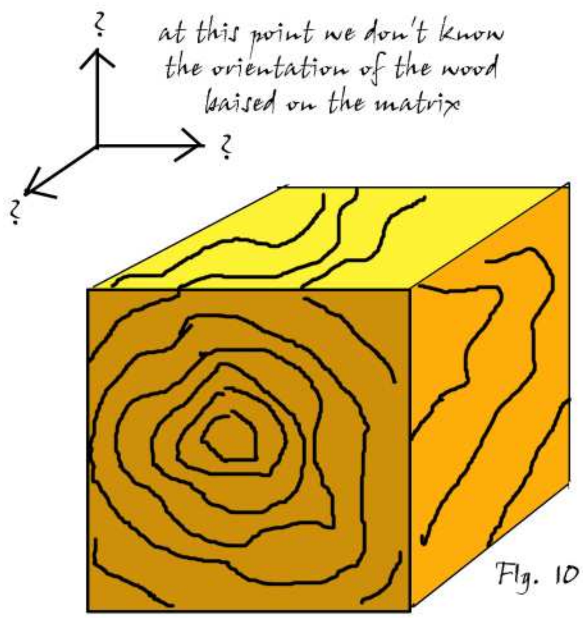


Fig. 10

The 17.004 value corresponds to the side of the tree. (right side in Fig. 10)

This insight is gained by the experiment of chopping down a tree. When chopping a tree it is important to angle the axe. If the axe is swung in perpendicular to the trunk the blade only compress the wood a little, but no cutting occurs. If the axe is angled it begins to guide itself parallel to the trunk and shears of a larger chunk (Fig.11). This is the .16 in the shear part of the diagonal.

Fig.11

When planning a board (making it flat) the easiest directions to go in are in a direction that rides along the growth rings. Recall each ring is a cylinder and we want to move along the longitudinal direction. This corresponds to the direction in and out of the paper in Fig. 10

Using this experimental data we can now orient the matrix with our wood. (Fig. 12)

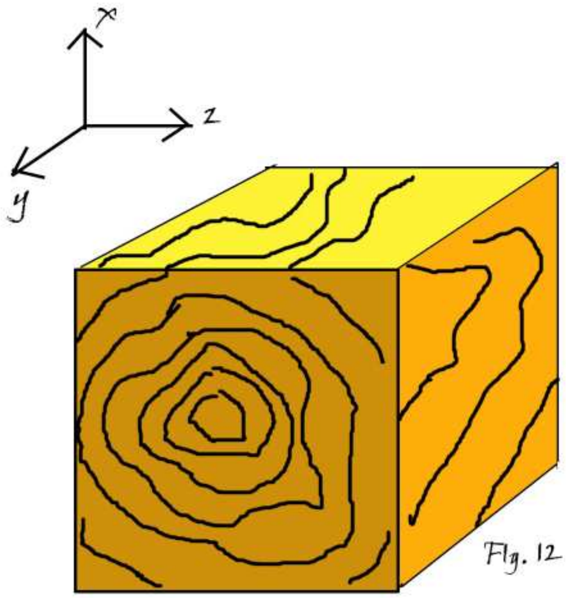


Fig. 12

In Conclusion we have found that a martial artist exploits the natural qualities of wood by selecting the side that is most likely to break (orthotropic). Utilizing physics in the way the strike is aimed to achieve a maximum speed. These facts along with training to maximize the ability to focus on the movements and task at hand allow the martial artist to perform an amazing feat of skill.

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All video provided by Korean Academy of Tae Kwon Do.