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Game Theory
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Game Theory and Olympic Tae Kwon Do Part 2 non-zero sum

Recall form the previous exploration of game theory and tae kwon do that zero sum games were an inadequate model. The main problem came with the fact Tae Kwon Do matches are won by the player with the most points, not by a balance of gains and losses. For a Zero sum game to work well the points of one player would have to be negative points for the other player. As we have seen looking at the net difference of the points received by the players does not work out well. Another problem with the earlier methods was the assumption that both players were equally matched, thus the constant tie as an equilibrium should not have been a surprise. With the zero sum matrix it would be computationally difficult to express different skill levels of the two players. So it is a non zero sum game looks to be a better way to go. Going back to the simpler matrix (Head body and defend) in a non-zero sum matrix gives us.

		Blue		
		defend	head	body
Red	Defend	0,0	0,0	0,0
	Head	0,0	2,2	2,1
	Body	0,0	1,2	1,1

If we now eliminate dominated strategies and look for a Nash equilibrium we find, that ore situation has not improved.

		Blue head	Blue body
	Head	2, 2	2, 1
Red	Body	1, 2	1,1

That is the equilibrium is the same one we found with the zero sum matrix, both players only kick to the head. Like the zero sum matrices form before adding more complex maneuvers will only result in both players adopting the “best” strategy. The end result of such an “arms race” will only lead back the problem that occurred with the zero sum models. Clearly more sophistication is called for. Some factors to consider are:

- Different skill levels
- Preference of types of kicks
- Speed (acceleration)
- Aggressiveness
- Probability of a technique scoring
- Flexibility
- Reflexes
- Range
- Timing
- Trapping

Different skill levels are self explanatory. Preference of kicks is a reference to the fact every tae kwon do player has a favorite kick and a

kick they are not to good at. Speed/acceleration is the ability of a player to go from a resting position to an attack quickly. Aggressiveness is the willingness to engage an opponent. A less aggressive player will have less opportunity to score. A flexible player will be able to kick an opponent at surprisingly close ranges, thus improving there chances to score on a less flexible player. A player with good timing can launch a successful attack that is quicker than his/her opponent's initial attack. Trapping is a bluff or fake attack to get an opponent to commit to a counter attack (see timing), then the trapping player switches to a different surprise attack.

For the new matrix the concepts of countering and trapping have been added to the previous matrix.

		Blue				
		Block	Head	Body	counter	trap
Red	Block	0, 0	0, 0	0, 0	n/a	0, 1
	Head	0, 0	2, 2	2, 1	0, 1	n/a
	Body	0, 0	1, 2	1, 1	0, 1	n/a
	counter	n/a	1, 0	1, 0	n/a	0, 1
	trap	1, 0	n/a	n/a	1, 0	n/a

In this matrix n/a has been entered since countering a block does not seem logical, etc…

Still there is no equilibrium in this matrix, more information is needed to find a solution.

Using a utility function to "encode" player information appears to be the optimal path to an answer. Building such a function is the tricky part, since every player will have their own personal function.

It is interesting to note that any prior knowledge of an opponent's "utility function" would be beneficial to a player. This kind information gathering does tack place at tournaments. Players find out who they will be sparring against and watch them in their other matches. For illustration purposes we will come up with two different functions to represent the differences of two players, Red and Blue. In this example we will only use the five categories from our earlier matrix; Block (D), Head (H), Body (B), Counter (C), and Trap (T). Even though only five categories appear many items form the list of things to account for can be "encoded" i.e. a player's flexibility will affect the effectiveness and desire to kick to the head. First up is Blue player whose traits are: more defensive, fast blocks, beginner skill level, and medium flexibility. This player's utility function is something like; u(Blue)=(.6)D+(.05)H+(.17)B+(.1)C+(.08)T
This function shows a preference tword defence, inexpirence in countering and traping, poor ability to kick to the head and moderate ability to score on the body.

Red player's traits are: more aggressive, fast kicks, advanced skill level, and excellent flexibility. A good function for Red is;
u(Red)= (.05)D+(.35)H+(.2)B+(.2)C+(.2)T
Due to Red's experience with trapping and countering and ability to scorn on the head well this player has little use for defense. The best defense is a good offence is this player's philosophy.

"Multiplying" the matrix by these functions yields the following.

		Blue				
		Block	Head	Body	counter	Trap
	Block	0, 0	.05, .1	.05, .17	.05, .1	0, .08
	Head	.7, .6	.7, .1	.7, .17	.7, .1	.7, .08
Red	Body	.35, .6	.2, .1	.2, .17	.2, .1	.35, .08
	counter	.2, .6	.2, .1	.2, .17	.2, .1	.2, .08
	trap	.2, 0	.2, .1	.2, .17	.2, .1	.2, .08

Note the purple highlighted cells are not (0,0) as before, Since the utility function is accounting for the success of a technique along with other factors these entries have been change to show in a contest between the players, which player would be more successful. i.e. in the head/block cell we have (.7,.6) Red player has an estimated point value of .7 points, the .6 of Blue shows his/her potential of blocking reds attack. The difference of these is .1 to Red showing Red can expect blue to block an attack enough of the time to bring the estimated point value down to .1.

The yellow cells have also changed for n/a to various values which show the potential success of the player's with these different techniques. These were added in to account for events like Blue tries to trap Red but Red is to fast and scores on blue before s/he ahs the chance to spring the trap. Now with this new and improved matrix we can look for a Nash equilibrium.

		Blue				
		Block	Head	Body	counter	trap
	Block	0, 0	.05, .1	.05, .17	.05, .1	0, .08
	Head	.7, .6	.7, .1	.7, .17	.7, .1	.7, .08
Red	Body	.35, .6	.2, .1	.2, .17	.2, .1	.35, .08
	counter	.2, .6	.2, .1	.2, .17	.2, .1	.2, .08
	trap	.2, 0	.2, .1	.2, .17	.2, .1	.2, .08

We now have found an equilibrium, Red should kick to the head and Blue should block Red's attacks. This solution also agrees with the

intuitive “solution” biased on the strengths of the two players. Larger and more “accurate” functions can be constructed and will probably give better and more dynamic “solutions” The more information put in to a utility function the longer and more complex the process of constructing the function. This is time most players would rather spend on training. But a good coach will do the “spying on other players and put together a “function” for the player s/he is coaching.

In the game of Tae Kwon Do sparring it is possible to reach an equilibrium and “solve” for the best strategies of the players involved. However it seems that in order to get to an equilibrium a lot of research and assumptions need to be made about the players that are matched up. Since every player will have his/her own utility functions it is an intensive process to plan for all opponents. This is the realm of a good coach. Most of all Tae Kwon Do players should practice and enjoy the sport.